INFORMATION TRANSFER IN QUANTUM MEASUREMENTS: IRREVERSIBILITY AND AMPLIFICATION *

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[Heisenberg] ... remarks ... that even in the case of macroscopic phenomena we may say, in a certain sense, that they are created by repeated observations ...

[Niels Bohr[1], in 1928 Nature article]

I. INTRODUCTION

The aim of these lectures is to investigate the transfer of information occurring in course of quantum interactions. In particular, I shall explore circumstances in which such an information transfer with the quantum environment of the considered quantum system leads to the destruction of the phase coherence between the states of the privileged basis in the system Hilbert space. This basis shall be called the pointer basis. I shall argue that states of this pointer basis correspond to the "classical" states of the observables of the quantum system in question.

The pointer basis emerges as a consequence of the environment-induced superselection rules: the system cannot be observed in a superposition of the states of the pointer basis, because the environment continuously performs a non-demolition measurement of the observable diagonal in the pointer basis.

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Applied to the quantum apparatus, the environment-induced superselection rules single out a preferred basis of the apparatus pointer. The apparatus can be observed only in one of the eigenstates of the pointer basis. This fact provides a natural explanation of the apparent reduction of the state vector. The increase of entropy associated with the process of reduction is then a consequence of the transfer of information into the environment degrees of freedom. Amplification of the measurement outcome facilitates this effectively irreversible information transfer and increases reliability of the correlation between the states of the pointer basis and the state of the measured system.

II. TWO STAGES OF THE MEASURENENT

The state vector of the measured quantum system becomes, as a result of the ideal measurement, one of the eigenstates of the measured operator. The choice of the particular, single eigenstate $|i\rangle$ and of the corresponding eigenvalue λ_i appears to be fundamentally stochastic. It is governed by the thoroughly tested probabilistic law

$$\{\text{probability of } |s\rangle \text{ to become } |i\rangle\} = p(|s\rangle \rightarrow |i\rangle) = |\langle s|i\rangle|^2 \ .$$

Yet the evolution equation for the state vector, e.g. the Schrödinger equation in nonrelativistic quantum mechanics, is linear and its solutions are deterministic and satisfy the principle of superposition. As a result the inherently random process of measurement culminating in the "collapse of the wave function" or "reduction of the state vector" cannot be fitted within the dynamical evolution of a closed quantum system. This discord between the apparently stochastic course of measurements on the one hand and the deterministic evolution of the state vectors on the other has been in the center of attention since the early days of the modern quantum theory. This is hardly surprising, as the process of measurement is of great importance by itself. Moreover, the random nature of the collapse allows one to expect that understanding it will shed a new light on the long-standing problems of statistical mechanics concerning irreversibility and the concept of entropy. This in turn should explain how the counterintuitive quantum phenomena can conspire to create the everyday classical, macroscopic reality.

The goal of this paper is to explore the idea that the measuring apparatus as well as other classical, macroscopic systems violate the quantum superposition principle because under normal conditions their observables interact with the external universe. Therefore, macroscopic systems cannot be regarded as "closed". To argue this I shall show how the effective superselection rules can be induced by the environment: "Monitoring" of the state of the system by the environment will be shown to result in the apparent collapse of the wave packet. Much of the discussion given below is an elaboration of the recently introduced idea of the pointer basis. It should be noted that the idea of macroscopic observables arising somehow in an open system observed by the environment has been recently expressed by Zeh[8] as well as by Wigner[9].‡

I shall begin the discussion of the measurement process by reviewing von Neumann's approach[3], who, in his analysis of the measurement, distinguishes two fundamentally different stages:

Establishment of the Correlation

During this first stage the state vector of the quantum apparatus, initially $|A_o\rangle$, becomes correlated with the state vector $|\psi\rangle$ of the measured system

$$|\Phi_i\rangle = |A_o\rangle \otimes |\psi\rangle = \left(\sum_n \alpha_n |A_n\rangle\right) \otimes \left(\sum_s c_s |s\rangle\right) \longrightarrow$$

$$\longrightarrow \sum_s c_s |A_s\rangle \otimes |s\rangle = |\Phi_f\rangle. \tag{2.1}$$

Here $\{|A_s\rangle\}$ and $\{|s\rangle\}$ constitute orthonormal bases in the apparatus and system Hilbert spaces, respectively. Such correlation can be accomplished by an appropriate interaction hamiltonian H^{AS} acting over a time interval τ:

$$|\Phi_f\rangle = exp(-iH^{AS}\tau/\hbar)|\Phi_i\rangle = U|\Phi_i\rangle. \tag{2.2}$$

If the system in the eigenstate of some operator $\hat{S} = \sum_s \lambda_s |s\rangle\langle s|$ is not to be perturbed by the measurement, H^{AS} with $\hat{S}$:

$$\left[H^{AS}, \hat{S}\right] = 0 \quad . \tag{2.3}$$

‡See, in particular, p. 268 of Ref. 8 and Eq. (27) of Ref. 9.

Above commmutation relation must be always satisfied by such "nonperturbative" or "nondemolition" measurements,[11–15] which will be of crucial importance for our further discussion.

The most important feature of a successful measurement—a one-to-one correlation between the state of the apparatus and the state of the measured system—is already present in the wave-function given by Eq. (2.1). There is, however, a fundamental difficulty caused by the fact that the final state given by that equation is pure, and not the appropriate mixture. This difficulty is best appreciated when we construct the density matrix corresponding to the final pure state $|\Phi_f\rangle$;

$$\rho_{pure} = |\Phi_f\rangle\langle\Phi_f| = \sum_s \sum_{s\prime} c^*_{s\prime} c_s |A_s\rangle\langle A_{s\prime}| \otimes |s\rangle\langle s\prime| \quad , \tag{2.4}$$

and compare it with the mixture diagonal in $|A_s\rangle\langle A_s| \otimes |s\rangle\langle s|$:

$$\rho_{mix} = \sum_s |c_s|^2 |A_s\rangle\langle A_s| \otimes |s\rangle\langle s| \tag{2.5}$$

It is this ρ_{mix} which contains probabilities of different outcomes, $|A_s\rangle\langle A_s| \otimes |s\rangle\langle s|$, with the appropriate weights. The density matrix ρ_{pure} contains, apart from the diagonal terms which are identical with ρ_{mix}, also off-diagonal "correlations". Their presence results in different physical properties of the ensembles described by ρ_{pure} and ρ_{pure} In particular:

(i) The state described by ρ_{pure} can be brought back to the initial state, $|A_o\rangle \otimes |\psi\rangle$, by means of hamiltonian evolution.

(ii) The apparatus contains not only complete information about the measured observable $\hat{S}$, but also information about other observables which do not commute with $\hat{S}$. This can be exhibited by rewriting $|\Phi_f\rangle$ in terms of an alternative orthonormal apparatus basis:

$$|\Phi_f\rangle = \sum_s c_s |A_s\rangle \otimes |s\rangle = \sum_p b_p |A_p\rangle \otimes |p\rangle \quad . \tag{2.6}$$

Here $|A_p\rangle = \sum_s \langle A_s|A_p\rangle |A_s\rangle$. The relative states[16] $|p\rangle = (b_p)^{-1} \sum_s \langle A_p|A_s\rangle |s\rangle$ are normalized, but, in the generic case, not necessarily orthogonal. They can nevertheless be used as "eigenstates" to define an observable which is

non-hermitean, but has many of the properties of hermitean operators.[17] Therefore, it is not clear what is the state vector in which the system has been registered by the apparatus. The ensuing problems are particularly apparent when all the coefficients c_s in Eq. (2.1) happen to be equal, and thus the set $\{|p\rangle\}$ of the relative basis states is orthonormal. Then the apparatus correlated with the measured system contains all the information not only about the observable $\hat{S}$, but also about the observable $\hat{P} = \sum_p \gamma_p |p\rangle\langle p|$. This is so despite the fact that $\hat{S}$ and $\hat{P}$ do not commute! Yet we know that quantum mechanics prevents one from simultaneously measuring two noncommuting observables. The quantum apparatus seems to ignore this law, following directly from the principle of indeterminacy! Furthermore, everyday experience convinces us that the choice of what the apparatus has measured cannot be made by us, arbitrarily long after the apparatus-system interaction has ceased, as the reasoning above seems to imply. To eliminate such difficulties von Neumann has postulated the occurrence of a second stage of the measurement process:

Reduction of the State Vector

During this second stage of the measurement process the density matrix ρ_{pure} of the combined apparatus-system object undergoes the transition:

$$\rho_{pure} = \sum_s \sum_{s\prime} c^*_{s\prime} c_s |A_s\rangle\langle A_{s\prime}| \otimes |s\rangle\langle s\prime| \longrightarrow$$

$$\longrightarrow \sum_s |c_s|^2 |A_s\rangle\langle A_s| \otimes |s\rangle\langle s| = \rho_{mix} \quad . \tag{2.7}$$

It is usually assumed that both the states of the apparatus and the states of the system are mutually orthonormal. However, as we have already discussed, states of the system relative to the privileged pointer basis states of the apparatus are, in general, not necessarily orthogonal.[17]

Reduction of the state vector cannot be accomplished in a closed system by means of unitary evolution. This is a straight forward consequence of the fact that unitary transformations evolve state vectors into state vectors. Hence, they are inherently incapable of transforming ρ_{pure}, a projection operator:

$$(\rho_{pure})^2 = \rho_{pure} \tag{2.8}$$

into ρ_{mix}, for which:

$$Tr\left(\rho_{mix}\right)^2 < Tr\rho_{mix} \tag{2.9}$$

To accomplish such transformation, one must employ a four-index object called "superoperator". Needless to say, objects like that cannot exist in the dynamics governed by Schrödinger equation. (Their existence has been nevertheless considered both in the context of statistical mechanics[18] and, more recently, in the field of black hole thermodynamics.[19,20])

Von Neumann was of course well aware of this dilemma. He favored the view according to which the conciousness of the observer was the ultimate cause of the collapse. This view was subsequently elaborated by London and Bauer[7] and revived by Wigner.[21] I shall explore an approach which seeks the explanation of the reduction of the wave packet in the fact that the apparatus, and, for that matter, any "classical" system is open, not closed. The goal of the quantum theory of measurement—accounting for the transition $\rho_{pure} \rightarrow \rho_{mix}$, Eq. (2.7)—is realized by the interaction of the apparatus with the environment. As a consequence correlation term of the pure state density matrix are damped away. Thus, the system cannot be observed in a superposition of the states that appear on the diagonal of ρ_{mix}: This can be interpreted by saying that there are effective superselection rules induced by the apparatus-environment information transfer.

III. ENVIRONMENT INDUCED SUPERSELECTION RULES

The purpose of this section is to demonstrate on an exactly soluble example how the interaction of a quantum system with its environment may induce superselection rules. I shall begin by investigating an interaction of a pair of two state systems—a bit by bit "measurement". Next I shall demonstrate how a single atom "environment", performing a non-demolition measurement on the apparatus atom can bring about the second stage of the measurement process. Finally, I shall show how the interaction with the many atom environment can induce superselection rules and single out the preferred pointer basis. This same mechanism for inducing superselection rules can operate in "real world" situations.

Bit-by-Bit "Measurement"

Consider a pair of two-state systems. I shall call one of them "spin" and the other of them "atom". Basis states of the spin can be denoted by $|\uparrow\rangle$, $|\downarrow\rangle$. Alternative basis can then be written as:

$$|\odot\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2} \quad , \tag{3.1a}$$

$$|\otimes\rangle = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2} \quad , \tag{3.1b}$$

or;

$$|\rightarrow\rangle = (|\uparrow\rangle + i|\downarrow\rangle)/\sqrt{2} \quad , \tag{3.2a}$$

$$|\leftarrow\rangle = (|\uparrow\rangle - i|\downarrow\rangle)/\sqrt{2} \quad , \tag{3.2b}$$

Basis states of the atom consist of the "ground state" $|\mp\rangle$ and the "excited state" $|\pm\rangle$. Despite this nomenclature, I shall assume that the energy of the "atom" in either of these states is identical. Alternative basis states of the atom are then:

$$|+\rangle = (|\pm\rangle + |\mp\rangle/\sqrt{2} \quad , \tag{3.3a}$$

$$|-\rangle = (|\pm\rangle - |\mp\rangle/\sqrt{2} \quad , \tag{3.3b}$$

and;

$$|_\top\rangle = (|\pm\rangle + i|\mp\rangle/\sqrt{2} \quad , \tag{3.4a}$$

$$|_\perp\rangle = (|\pm\rangle - i|\mp\rangle/\sqrt{2} \quad , \tag{3.4b}$$

To investigate the process of the transfer of information consider an interaction between the atom and the spin given by the hamiltonian:

$$\begin{aligned} H^{AS} &= g\,(|_\perp\rangle\langle_\perp| - |_\top\rangle\langle_\top|) \otimes (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) \\ &= ig\,(|\pm\rangle\langle\mp| - |\mp\rangle\langle\pm|) \otimes (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) \quad . \end{aligned} \tag{3.5}$$

Suppose that the initial state of the atom is $|+\rangle$ and the initial state of the spin is $a|\uparrow\rangle + b|\downarrow\rangle$: hence the combined state vector $|\Phi\rangle$ is initially given by the direct product:

$$|\Phi_i\rangle = |\Phi(0)\rangle = |+\rangle \otimes (a|\uparrow\rangle + b|\downarrow\rangle) \quad . \tag{3.6}$$

As a result of the hamiltonian evolution generated by H^{AS} $|\Phi\rangle$ is transformed into:

$$|\Phi(t)\rangle = a|\uparrow\rangle \otimes [\sin(\pi/4 + gt)|\pm\rangle + \cos(\pi/4 + gt)|\mp\rangle]$$

$$+b|\downarrow\rangle \otimes [\sin(\pi/4 - gt)|\pm\rangle + \cos(\pi/4 - gt)|\mp\rangle] \quad . \tag{3.7}$$

Above, and throughout most of this paper I shall set; $\hbar = 1$. Consider now the state of the combined atom-spin system when $t = \pi/4g = \tau$. Then:

$$|\Phi_f\rangle = |\Phi(\tau)\rangle = (a|\uparrow\rangle \otimes |\pm\rangle + b|\downarrow\rangle \otimes |\mp\rangle)/\sqrt{2} \quad . \tag{3.8}$$

This is still, beyond any doubt, a pure state. However, both the spin and the atom, when treated as individual systems, are in mixed states. This can be easily shown by calculating their density matrices:

$$\rho^S = |a|^2|\uparrow\rangle\langle\uparrow| + |b|^2|\downarrow\rangle\langle\downarrow| \quad , \tag{3.9a}$$

$$\rho^A = |a|^2|\pm\rangle\langle\pm| + |b|^2|\mp\rangle\langle\mp| \quad . \tag{3.9b}$$

The off-diagonal terms vanish. This accomplishes the apparent collapse for each of the subsystems described by the total, still pure wave function $|\Phi(\tau)\rangle$.[§]

Environment-Induced Superselection Rules

Having concluded the first stage of the measurement—an establishment of the correlation between the apparatus-atom and the spin-system, Eq. (3.8)—one may ask if there is any reason to worry about the reduction of the state vector. After all, perfect correlation implies that by consulting the atom and "reading off" its state (either $|\pm\rangle$ or $|\mp\rangle$) one can find out whether the spin is $|\uparrow\rangle$ or $\downarrow\rangle$— parallel or antiparallel to the Z-axis. What more can one ask from a perfect measeurement? To expose the difficulty associated with the purity of $|\Phi(\tau)\rangle$ let us rewrite Eq. (3.8) using $|+\rangle$, $|-\rangle$ as a basis for the atom Hilbert space. For simplicity, we consider the case when $a = b = 1/\sqrt{2}$. Then:

$$\begin{aligned} |\Phi(\tau)\rangle &= (|\uparrow\rangle \otimes |\pm\rangle + |\downarrow\rangle \otimes |\mp\rangle)/\sqrt{2} \\ &= (|\odot\rangle \otimes |+\rangle + |\otimes\rangle \otimes |-\rangle)/\sqrt{2} \quad . \end{aligned} \tag{3.9}$$

Thus, by "looking" at the apparatus in the different basis one can find out whether the spin is parallel or antiparallel to the X-axis! And the decision on how (in what basis) shall one measure the state of the atom and, hence, in what direction will the spin eventually point can be made long after the

[§]A thought experiment which realizes the transition from the initial $|\Phi(0)\rangle$ given by Eq. (3.6) to the final $|\Phi(\tau)\rangle$ of Eq. (3.8) employing the reversible Stern-Gerlach apparatus has been recently described by Scully, Shea and McCullum[22] and Scully[23].

spin-atom interaction has ceased! This affords another example of the counterintiutive "delayed-choice" features of quantum phenomena, first discussed by Wheeler[24,25] and intimately connected with the so-called "nonseparability". Certainly, no real world measuring apparatus leaves this delayed-choice option open for the experimenter. And the purity of $|\Phi(\tau)\rangle$ is the origin of this surprising feature.¶

Information transfer from the systems (atom, spin) into correlations between them was the ultimate source of the apparent collapse of the state vector of both spin and atom. Can this mechanism of information transfer be employed to bring about the transition from ρ_{pure} to ρ_{mix} for the combined apparatus-system object? To see how, let us begin with the pure wave-function $|\Phi(\tau)\rangle$, given by Eq. (3.8). Let us suppose that the atom plays the role of the apparatus and the spin is the measured system. To remove the off-diagonal terms from the matrix;

$$\rho^{AS}_{pure} = |\Phi(\tau)\rangle\langle\Phi(\tau)| = |a|^2|\pm\rangle\langle\pm| \otimes |\uparrow\rangle\langle\uparrow| + |b|^2|\mp\rangle\langle\mp| \otimes |\downarrow\rangle\langle\downarrow|$$

$$+ab*|\pm\rangle\langle\mp| \otimes |\uparrow\rangle\langle\downarrow| + a*b|\mp\rangle\langle\pm| \otimes |\downarrow\rangle\langle\uparrow| \quad , \tag{3.10}$$

we can couple the apparatus-atom with the two-state "environment atom". The states of the environment atom are designated by parentheses rather than Dirac's brackets; $|\pm)$, $|\mp)$; $|+)$, $|-)$; $|_\perp)$, $|_\top)$, respectively. The coupling hamiltonian is:

$$H^{AE} = g_1(|_\perp)(_\perp| - |_\top)(_\top|) \otimes (|\pm\rangle\langle\pm| - |\mp\rangle\langle\mp|)$$

$$= ig_1(|\pm)(\mp| - |\mp)(\pm|) \otimes (|\pm\rangle\langle\pm| - |\mp\rangle\langle\mp|) \quad . \tag{3.11}$$

When the initial state of the environment is $|+)$, i.e., the initial state of the combined apparatus-system environment is:

$$|\psi_i\rangle = (a|\uparrow\rangle \otimes |\pm\rangle + b|\downarrow\otimes|\mp\rangle) \otimes |+) \quad . \tag{3.12}$$

Then the final state for $g_1 t = \pi/4$ will be:

$$|\psi_f\rangle = (a|\uparrow\rangle \otimes |\pm\rangle \otimes |\pm) + b|\downarrow\otimes|\mp\rangle \otimes |+\rangle) \quad . \tag{3.13}$$

¶The relation between the nonseparability, complemntarity and delayed-choice was further discussed on the example of Einstein's version of double slit experiment by Wootters and Zurek[26]. A practical realization of such experiments has been discussed by Wickes, Alley and Jakubowicz as well as by Bartell[27].

When the states of the environment atom are traced out, one obtains a final state density matrix:

$$\rho^{AS}_{mix} = |a|^2|\uparrow\rangle\langle\uparrow| \otimes |\pm\rangle\langle\pm| + |b|^2|\downarrow\rangle\langle\downarrow| \otimes |\mp\rangle\langle\mp| \tag{3.14}$$

This shows how, employing partial information transfer from the apparatus to the environment, one can accomplish the transition $\rho_{pure} \rightarrow \rho_{mix}$ required for the collapse of the state vector, Eq. (2.7).

It is important to distinguish essential and inessential requirements for the occurrence of the environment induced collapse. One may, for instance, object that for the real world environments conditions of terminating the interaction at $g_1 t = \pi/4$ or starting from a particular, pure initial state are artificial and hence the proposed solution is not applicable to the measurement problem. To counter these objections I shall investigate a more realistic case when the environment consists of many noninteracting atoms, each of them coupled to the apparatus atom by the hamiltonian H^{AE}_k:

$$\begin{aligned} H^{AE}_k &= ig_k(|\pm)(\mp| - |\mp)(\pm|)_k \otimes (|\pm\rangle\langle\pm| - |\mp\rangle\langle\mp|) \prod_{j\neq k} \otimes 1_j \\ &= -g_k(|_\perp)(_\perp| - |_\top)(_\top|)_k \otimes (|\pm\rangle\langle\pm| - |\mp\rangle\langle\mp|) \prod_{j\neq k} \otimes 1_j \end{aligned} \tag{3.15}$$

then the resulting wave function $j \neq k$, at an arbitrary time t, is:

$$\begin{aligned} |\psi(t)\rangle = a|\uparrow\rangle \otimes |\pm\rangle \prod_{k=1}^{N} \otimes [\alpha_{k\uparrow}(t)|_\perp) + \beta_{k\uparrow}(t)|_\top)] \\ + b|\downarrow\rangle \otimes |\mp\rangle \prod_{k=1}^{N} \otimes [\alpha_{k\downarrow}(t)|_\perp) + \beta_{k\downarrow}(t)|_\top)] \end{aligned} \tag{3.16}$$

Here N is the total number of the environment atoms, and:

$$\alpha_{k\uparrow}(t) = \alpha_k \exp(-ig_k t); \quad \alpha_{k\downarrow}(t) = \alpha_k \exp(+ig_k t)$$

$$\beta_{k\uparrow}(t) = \beta_k \exp(+ig_k t); \quad \beta_{k\downarrow}(t) = \beta_k \exp(-ig_k t)$$

Constants α_k and β_k are fixed by the initial states of the environment atoms. Now one can calculate the density matrix describing the system-apparatus combination by taking the partial trace over the states of the environment:

$$\rho^{AS} = |a|^2 |\uparrow\rangle\langle\uparrow| \otimes |\pm\rangle\langle\pm| + |b|^2 |\downarrow\rangle\langle\downarrow| \otimes |\mp\rangle\langle\mp|$$

$$+z(t)ab^* |\uparrow\rangle\langle\downarrow| \otimes |\pm\rangle\langle\mp| + z^*(t)a^*b |\downarrow\rangle\langle\uparrow| \otimes |\mp\rangle\langle\pm| \quad . \tag{3.17}$$

Above the correlation-damping factor $z(t)$ is given by:

$$z(t) = \prod_{k=1}^{N} \left[\cos 2g_k t + i\left(|\alpha_k|^2 - |\beta_k|^2\right) \sin 2g_k t\right] \tag{3.18}$$

When the absolute value of the correlation-damping factor $z(t)$ is much less than unity, the off-diagonal terms of the density matrix effectively disappear. The correlations between the apparatus and the measured system are, nevertheless, left intact in the preferred pointer basis $[|\pm\rangle,\ |\mp\rangle]$ of the apparatus.

Information Conservation and the Irreversible Decay of Correlations

It is not difficult to show, that $z(t)$ starting initially ($t = 0$) at $z(0) = 1$, will quickly fall off to values $z(t) \simeq 0$. In particular, when the coupling constants $\{g_k\}$ are chosen at random one can readily show that:

(i) $\quad \langle z(t) \rangle = 0$;

(ii) $\quad \langle |z(t)|^2 \rangle = \prod_{k=1}^{N} [(1 + \gamma_k^2)/2]$;

where $\gamma_k = ||\alpha_k|^2 - |\beta_k|^2|$.

Thus, unless $\gamma_k = 1$ for all k (the environment atoms are in the eigenstates of the apparatus-environment interaction hamiltonian) the information transfer from the apparatus to the environment will cause the effective collapse since:

$$\left(\langle |a(t)|^2 \rangle\right)^{\frac{1}{2}} << 1 \quad . \tag{3.19}$$

A typical plot of $z(t)$ for three environments of increasing size ($N = 5, 10, 15$) is shown in Figure 1. It is apparent there that even a relatively small environment of $N = 15$ is capable of making $z(t) \simeq 0$ for a very long time.

THIS FIGURE IS ESSENTIALLY IDENTICAL WITH THE ILLUSTRATION OF THE CORRELATION-DAMPING FACTOR IN W. H. Zurek, *Environment - Induced Superselection Rules*, Physical Review **D26**, 1862 (1982). **THE ORIGINAL CAN BE OF COURSE FOUND IN THE PROCEEDINGS, SEE THE FOOTNOTE ON THE VERY FIRST PAGE OF THIS PAPER**

Figure 1: Correlation-damping factor $z(t)$, see Eq. (3.17) and Eq. (3.18), for the spin in a two-state atom environment. Number of the environment atoms is, respectively, (a) $N = 5$; (b) $N = 10$; (c) $N = 15$. Atom-spin coupling constants g_k of Eq. (3.18) were chosen randomly with the distribution uniform in the open interval $(0, 1)$. For simplicity the initial conditions $|\alpha_k| = |\beta_k|$ was imposed for all atoms, and, consequently, $z(t)$ is purely real. Note disappearance of the recurrences with the increase of N.

It is, however, also clear that the total state of the combined, apparatus-system-environment object evolves in a unitary fashion, and no information is really irreversibly destroyed. The decay of $z(t)$ is due to the transfer of information, and not due to the destruction of information. Thus, if the environment of the real world apparata were an isolated, closed system, then after a sufficiently long period of time T the value of $z(t)$ would have to return back arbitrarily close to the initial $z = 1$. In particular, when all g_k are commensurable, $z = 1$ would recur exactly. The recurrence of correlations encountered here is closely reminiscent of the Poincaré recurrence cycle of classical mechanics.[41]

One could expect, that the ultimate "decay of correlations" and the disappearance of recurrences could take place in the limit of large N. This reasoning, often adopted in the discussions of irreversibility and quantum measurements[28–32] contains at best only a part of the collapse mechanism: it shows that in the hamiltonian evolution information can be transferred into those degrees of freedom which are deemed "not observable" due to some external restriction. The rationale behind this procedure is very similar to the old and well-known idea of "coarse graining". Taking limits $\tau \to \infty$ or $N \to \infty$ may help in finding out what are the macroscopic degrees of free-

dom, but provides no real clue as to why there is a collapse of the wave function; the total wave function is still pure. Moreover, neither of these limits is relevant in the case of real, and hence finite in size and duration, physical experiments.[‖] As long as the von Neumann (Schröinger) equation remains rigorously valid, there can be no rigorous collapse of the wave packet of the closed system.

The situation is quite different for open systems. When the information is transferred from the system to the environment, and the environment itself is also an open system—as in practice is always the case—then the information is "passed on" and continues spreading away from where it has originated. Evolution of the open system is no longer unitary. The effective collapse takes place not because of the thermodynamic or ergodic properties of the investigated system: it is rather a consequence of its interaction with the environment. Part of this idea is contained already in the "usual" derivations of the master equations for quantum systems by means of projection operators[34]. There the role of the environment is played by the correlation degrees of freedom. Moreover, difficulties of the usual derivation of an irreversible master equation, (see, for example, a discussion of Zeh[8]) e.g. the necessity of the repeated "random phase approximation" can be resolved when the role of the environment is properly taken into account: the environment itself continuously recollapses the state of the system.

IV. INFORMATION TRANSFER

The aim of this section is to provide a mathematical definition of the concept of transfer of information. I shall follow closely Everett's discussion[36] which employs Shannon's definition of information[37]. For a density matrix ρ in an N-dimensional Hilbert space information I is given by the formula:

$$I = \ln N - Tr(\rho \ln \rho) \quad . \tag{4.1}$$

It is well known that if ρ evolves according to von Neumann equation I is a constant of motion. I shall use information defined by Eq. (4.l) in the discussion of the information transfer between the spin and the atom in the bit-by-bit "measurement" described in the previous section. An extrapolation

[‖]This point was expressed particularly clearly by Bell[33] in his discussion of Hepp-Coleman model[30].

of the formalism presented below to more general situations is conceptually straightforward, even though it does encounter technical difficulties in the case of continuously infinite Hilbert spaces. Some of these difficulties have been already pointed out by Everett.[36]

Consider the initial wave function $|\Phi(0)\rangle$ of the combined spin-atom object, given by the direct product, Eq. (3.6).

$$|\Phi(0)\rangle = |+\rangle \otimes (a|\uparrow\rangle + b|\downarrow\rangle) \quad .$$

The total system as well as each of the subsystems is in a pure state. The total information can be then readily evaluated:

$$I^{AS} = \ln 4 - Tr\rho^{AS} \ln \rho^{AS} = \ln 4 \tag{4.2}$$

This total information can be expressed as a sum of two separate contributions, originating in two subsystems, the atom and the spin:

$$I_A = \ln 2 - Tr\rho^{A} \ln \rho^{A} = \ln 2 \tag{4.3a}$$

$$I_S = \ln 2 - Tr\rho^{S} \ln \rho^{S} = \ln 2 \tag{4.3b}$$

Consider now the final wave function:

$$|\Phi(\tau)\rangle = (a|\uparrow\rangle \otimes |\pm\rangle + b|\downarrow\rangle \otimes |\mp\rangle)$$

given by Eq. (3.8). Since the combined system is still in a pure state, the total information must be still $I^{AS} = \ln 4$, the same as before the establishment of the correlation. This is no longer true of the subsystems. The respective informations are, in general, less than $\ln 2$:

$$I^A = I^S = \ln 2 - \left(|a|^2 \ln |a|^2 + |b|^2 \ln |b|^2\right) \quad . \tag{4.4}$$

In particular, when $a = b = 1/\sqrt{2}$, $I^A = I^S = 0$. In the generic case

$$\mathcal{I} = I^A + I^S < I^{AS} \quad . \tag{4.5}$$

Where is the missing information I_c?

$$I_c = I^{AS} - \mathcal{I} \tag{4.6}$$

It does not take long to realize that I_c resides in the correlations between the systems, and not in the systems.

In terms of information transfer the bit-by-bit "measurement" in the particular case $a = b = 1/\sqrt{2}$ can be described by stating that the definiteness of the states of the separate systems has been transformed into the definiteness of the correlations between them. Or, to put it still differently, before the interaction between the spin and the atom their correlations with the observer (who has set up their initial states) were definite: After the interaction the correlations between the observer and the two subsystems disappear; they are replaced by the correlations between the subsystems. Of course, no information is lost: it is only transferred from one set of correlations to the other.

One can generalize the formalism of information transfer to arbitrary systems in finite-dimensional Hilbert spaces. Suppose the combined system under consideration consists of N subsystems, the kth subsystem being described by the n_k dimensional Hilbert space $\mathcal{H}_k$. The total Hilbert space is then given by the direct product:

$$\mathcal{H} = \prod_{k=1}^{N} \otimes \mathcal{H}_k \quad . \tag{4.7}$$

The total information I—a constant of motion under the evolution generated by the von Neumann equation—is given by:

$$I = \ln \left(\prod_{k=1}^{N} n_k \right) - Tr(\rho \ln \rho) \quad . \tag{4.8}$$

where ρ is the density matrix of the whole system. The information residing in the kth subsystem is given by:

$$I_k = \ln(n_k) - Tr \rho_k \ln \rho_k \tag{4.9}$$

where ρ_k, density matrix of the kth sybsystem, is obtained from ρ by a partial trace. The total information in the subsystems is equal to:

$$\mathcal{I} = \sum_{k=1}^{N} I_k \tag{4.10}$$

The total information in correlations between the subsystems is given by:

$$I_c = I - \mathcal{I} = I - \sum_{k=1}^{N} I_k \tag{4.11}$$

When $I_c = 0$ the subsystems are uncorrelated. In quantum measurements this situation occurs at the beginning of stage 1, before the correlations between the two subsystems of interest—the apparatus and the measured system—are set up. The first stage of measurement can be described as a transfer of the information from the observer-apparatus correlations to the apparatus-system correlations. After the completion of this stage the total information of the apparatus system combination is still the same as it was before their interaction has started. This excessive amount of correlation is responsible for the paradoxes associated with irreversibility and non-separability. To prevent the occurrence of these paradoxes von Neumann postulated the reduction of the state vector. In the course of this process the off-diagonal terms of the density matrix, which contain information about spurious correlations, are supposed to disappear. The key difficulty of the quantum theory of measurement is the incompatibility of the reduction postulate with the unitarity of the evolution equations. In other words, in a closed system it is impossible to get rid of this spurious information. The resolution considered here asserts that the spurious information is removed by the environment. Thus, it is never actually destroyed: we ignore it, because it isn't a part of our combined system anymore.

V. DISCUSSION

The aim of these lectures was to investigate the idea that the second stage of the measurement process—the apparent reduction of the state vector—can be accomplished through a transfer of a part of the information from the apparatus-system into the apparatus-environment correlations. All the evolutions are unitary: the information is ultimately never destroyed—it is only displaced. The pointer basis of the quantum system is subjected to a continuous measurement performed on it by the environment, which plays the role of an (unread) apparatus. As a result, effective superselection rules arise. Continuous destruction of the phase information causes the pointer observable to behave in a way which makes it appear as "classical". It is extremely important to stress that this classical behavior of the pointer observable is not a consequence of just the structure of the Hilbert space in

which it is defined: The exact pointer observable must commute with the total hamiltonian of the investigated system (i.e. the self-hamiltonian and the system environment interaction hamiltonian). Classical properties of a quantum system are induced by the "measurements" performed on it by the environment. In this sense one can claim that properties of the system originate in part in its interactions with the environment. The system is "the way it is" because it is perceived by the environment in a particular way. This conclusion is in contradiction with intuitions based on classical phenomena, and is a direct consequence of quantum mechanics.

Environment-induced superselection rules, counterintuitive as they may seem, answer many of the questions concerning the apparent collapse of the wave packet. There are, of course, many more questions they allow to be raised. The aim of this section is to ask—and, at least in part, answer—some of these questions.

The Density Matrix in the Presence of the Environment-Induced Superselection Rules

Bell[33] has once said that "In quantum measurement theory elimination of ... [the off-diagonal density matrix terms expressing] ... coherence is the philosophers stone". Selective information transfer resulting in the environment-induced superselection rules accomplishes this task by transferring spurious correlations into the environment. The first question that one can raise is whether transfer of correlations accomplishes the same goal as their destruction would. I shall come back to it later in this section. Here let me consider a simpler, but equally important problem and ask: what have we really gained by destroying correlations? Clearly, predictive power of quantum theory has been enhanced by our considerations, but only in a very special way: one can tell now what a given apparatus measures, but—as before—one cannot tell what the outcome of the measurement will be. The diagonal part of the density matrix is still the same, giving only probabilities of different possible outcomes of the measurement process. This is so despite the fact that when someone consults the apparatus the outcome he is going to see will be a definite eigenvalue of the measured operator—a definite eigenstate of the pointer observable. And we still do not know which eigenstate it is going to be. We have, nevertheless, learned into what mixture shall the combined apparatus-system state vector collapse. Hence we know what information

about the system still remains in the apparatus.

The aim of the discussion in this subsection is to explore the interpretation according to which a density matrix diagonal in the pointer basis describes, in presence of the environment-induced superselection rules, a quantum system which is in a definite state—in one of the pointer basis eigenstates. The probabilistic character of the density matrix results therefore, according to this point of view, solely from our ignorance of the true state of the system—and not from the fact that the system itself is not in a definite state.

This interpretation may seem obviously correct. For, let me suppose that my friend prepares an ensemble of spins, one by one, in states $|\uparrow\rangle$ or $|\downarrow\rangle$ with respective probabilities $\rho_\uparrow$ and $\rho_\downarrow$. Then one can maintain that any particular spin in this ensemble is in a well- defined state (i.e. my friend knows that he has prepared spin #137 in the state $|\uparrow\rangle$, etc.). However, if my friend refuses to share this detailed information with me and tells me only values of $\rho_\uparrow$ and $\rho_\downarrow$, I shall have to trace out the memory of my friend and represent each spin by a density matrix:

$$\rho_{spin} = \rho_\uparrow |\uparrow\rangle\langle\uparrow| + \rho_\downarrow |\downarrow\rangle\langle\downarrow| \quad . \tag{5.1}$$

In the situation described above the density matrix is then nothing else but the expression of my ignorance: I do not know in which of the eigenstates appearing on the diagonal ρ_{spin} particular spin (#137) was prepared.

Can one always apply this interpretation to any diagonal density matrix? Unfortunately not. To see why, let me suppose that $\rho_\uparrow$ ¿ $\rho_\downarrow$. Then ρ_{spin} can be rewritten, for instance as:

$$\rho_{spin} = (\rho_\uparrow - \rho_\downarrow)|\uparrow\rangle\langle\uparrow| + \rho_\downarrow(|\odot\rangle\langle\odot| + |\otimes\rangle\langle\otimes|) \quad . \tag{5.2}$$

That is, my friend could have prepared the spin ensemble by mixing spins in the state $|\uparrow\rangle, |\odot\rangle$ and $|\otimes\rangle$ in the appropriate proportions. In fact, there are continuously many ways in which he could have prepared the ensemble described by Eq. (5.1). And only in one of these ways the initial claim—that the system is in a well-defined state, either in $|\uparrow\rangle$ or in $\downarrow\rangle$—has any justification. In all the other instances, as our friend can testify, this claim is simply wrong.

This well-known[38] nonuniqueness of the ensemble described by a given

density matrix disappears in presence of the environment- induced superselection rules. The mixture is now diagonal in the uniquely specified pointer basis. The environment—which continuously measures systems under consideration—contains a record of their individual states. The troublesome Equation (5.2) can be of course written, but it cannot by physically realized. For, even if our friend will prepare some of the spins in the state $|\odot\rangle$ or $|\otimes\rangle$, or, for that matter, in any state other than one of the pointer basis states (which we can assume for the sake of the argument to be $\{|\uparrow\rangle, |\downarrow\rangle\}$), the state of each spin will almost immediately "collapse" to a mixture diagonal in the pointer basis. That is:

$$(a|\uparrow\rangle + b|\downarrow\rangle)(a^*\langle\uparrow| + b^*\langle\downarrow|) \rightarrow |a|^2|\uparrow\rangle\langle\uparrow| + |b|^2|\downarrow\rangle\langle\downarrow| \quad .$$

The record of our friend is no longer trustworthy—it is the environment which keeps the true record of what the spin is.

Therefore, one must conclude that in absence of the superselection rules the same density matrix describes, in the generic case, continuously many different ensembles. This nonuniqueness is removed by the environment-induced superselection rules: a density matrix diagonal in the pointer basis describes now a unique ensemble consisting of individual systems each of which can be thought of as being in the specific eigenstate of the pointer observable. This interpretation, tempting as it is, still faces a major problem: The environment has not destroyed the correlation information, and so one can presumably still reverse the evolution of the complete (i.e., including the environment) system to restore the initial wavefunction. I shall discuss this problem in the last subsection of this section.

Two Origins of the Entropy Increase

According to von Neumann the second stage of the measurement process is explicitly irreversible: the information is destroyed. In the model of the reduction of the state vector investigated here the information is only displaced. Can one consider this an irreversible process? And—assuming that the measurement theory proposed here reflects accurately the real act of measurement—does it help to understand the second law of thermodynamics? It is not yet possible to give a final, complete answer to these questions. The following few remarks have therefore a rather preliminary character.

Entropy and its definition is the content of the first remark. Let us examine $\mathcal{I}$, the total information I less the information residing in the correlations I_c,

$$\mathcal{I} = I - I_c$$

as a candidate for the negative of the entropy (modulo a constant). For simplicity, we shall call $\mathcal{I}$ "negentropy". As defined in the previous section, the negentropy $\mathcal{I}$ represents the information available to the observer about the subsystems of the physical system under investigation. Once these subsystems—i.e. the partition of the complete Hilbert space into subspaces—is fixed, the negentropy $\mathcal{I}$ is a well-defined quantity. It can be calculated directly from Eq. (4.9), (4.10). Of course, one may object to this definition of entropy by arguing that it shifts the subjective element from the usual and arbitrary "coarse-graining" to the equally arbitrary concept of the subsystem. This objection, correct as it is in classical statistical mechanics cannot be simply transplanted into the context of quantum mechanics. For, in the classical phase space shifting from one coarse-grained network of cells to the other can be achieved, in principle, at no expense in terms of energy: Classical mechanics does not recognize that any information transfer can be always ultimately reduced to an interaction. In quantum mechanics, on the other hand, partition of the original Hilbert space into smaller subspaces is no longer arbitrary: It depends on the structure of the coupling between the system and the measuring apparatus, the pointer basis of the apparatus, as well as on the hamiltonian of the investigated system. Therefore, the change of the concept of a subsystem is not just a purely "mental" act. On the contrary, it implies a change of the coupling hamiltonians and, presumably, requires an expenditure of a certain amount of energy. For example, in a system of N hydrogen molecules the natural "coarse-graining" will recognize molecules, atoms and elementary particles. The problem of determining what are the natural subsystems of a certain system from the complete hamiltonian is, of course, by no means trivial. I shall not attempt to solve it here. Some light has been thrown on it by the idea of subdynamics[39]. More recently, Deutsch[40] has considered the product structure induced on the Hilbert space by the Hamiltonian. His approach, in a sense complementary to the one adopted here, gives also equations for the preferred "interpretation basis", which is expected to play a role similar to the pointer basis discussed in this paper.

The apparent increase of entropy is the subject of the second remark. I shall argue that its increase on the relaxation time scale is a consequence of a mismatch between the division of a system into subsystems and the evolution generated by the self-hamiltonian of the considered system, H^S , which induces correlations between these subsystems. Apart from this short-term relaxation caused by the increase of the correlation information I_c on the expense of negentropy $\mathcal{I}$, with I constant, I shall also consider in the last part of this subsection long-time relaxation, occurring over periods of the order of the recurrence time of the system, and caused by the interaction with the environment and resulting in the decrease of I itself.

Suppose that the total Hilbert space of the considered system is partitioned into N subsystems, in a manner represented by a direct product, Eq. (4.7). I shall moreover suppose that one can measure arbitrarily accurately any observable that is confined to one of the rigidly defined N subsystems, corresponding to N subspaces $\mathcal{H}_k$. As a result of such an ideal measurement on the subspace $\mathcal{H}_k$ one obtains an ℓ^{th} eigenvector $|k_\ell\rangle$ of the measured observable $\hat{O}_k$. After all the measurements are completed, the state vector of the whole system is given by

$$|s_{P_0}\rangle = |\Phi(0)\rangle = \prod_{k=1}^{N} \otimes |k_\ell\rangle \quad . \tag{5.3}$$

I shall call each state which can be expressed in the form of the above product, with vectors $|k_\ell\rangle$ arbitrary but confined to the sub-spaces $\mathcal{H}_k$ a "product state". If the measurements within each $\mathcal{H}_k$ are exhaustive (i.e. all the commuting observables are measured) then the collection of all orthogonal products of the form of Eq. (5.3) provides a complete basis in the space $\mathcal{H}$. States spanning this basis can be numbered with an index P, referring to the permutation of the subscripts appearing on the right hand side of Eq. (5.3). Let us moreover suppose that the self-hamiltonian of the system, H^S, has a complete set of the eigenvectors $|\chi_j\rangle$:

$$H^S|\chi_j\rangle = \epsilon_j|\chi_j\rangle \tag{5.4}$$

In the generic case H^S will be degenerate, i.e. not all ϵ_j will be different. Now the general solution for the wave function which at $t = 0$ was given by Eq. (5.3) can be written as:

$$|\Phi(t)\rangle = \sum_{P,j} \left[\langle s_P|\chi_j\rangle \exp(-i\epsilon_j t)\langle\chi_j|\Phi(0)\rangle\right] |s_P\rangle = \sum_P \alpha_{P_0 P}(t)|s_P\rangle \quad . \tag{5.5}$$

Above evolutions can be classified into two categories, which I shall tentatively call "correlating" and "non-correlating". In the course of a non-correlating evolution the state vector $|\Phi(t)\rangle$ can be for all t re-expressed as a product state:

$$|\Phi(t)\rangle = \prod_{k=1}^{N} \otimes |k_\ell(t)\rangle \quad . \tag{5.6}$$

That is, the self-hamiltonian causes state vectors of the subsystems to rotate within their respective subspaces $\mathcal{H}_k$. This does not necessarily mean that the subspaces $\mathcal{H}_k$ are non-interacting: It suffices for the states $|k_\ell\rangle$ appearing in the product in Eq. (5.3) to be separately eigenstates of the hamiltonian H^S. For example, in the spin-atom interaction hamiltonian H^{AS} Eq. (3.5), states $|_\perp\rangle$, $|\downarrow\rangle$ had that property of not becoming correlated.

Correlating evolution, on the other hand, introduces into $|\Phi(\tau)\rangle$, Eq. (5.5) terms which cannot be reduced to the form of Eq. (5.6). Evolution of the initial state vector $|\pm\rangle \otimes |\odot\rangle$ under the influence of H^{AS}, Eq. (3.5) is an example. There are, moreover, interaction hamiltonians between the subsystems which introduce correlations into any intial state vector $|\Phi(0)\rangle$. For such hamiltonians** there is no eigenstate which can be decomposed into a product of the form of Eq. (5.3). The increase of entropy is then caused by the increase of correlations between the subsystems of the whole system. Moreover, each measurement brings the system into a product state with a product structure conforming with the division into the subsystems. Therefore, it decreases the entropy of the system with respect to the measuring device by destroying correlations between subsystems. Consequently, following such a measurement, one can expect that the natural evolution of the system will result in a buildup of the correlations and in the increase of entropy. It is perhaps worth pointing out that the view adopted here is close to the one of Boltzmann. The key to the understanding of the entropy increase lies, according to these proposals, in the distinction between the information about the subsystems, which I have called negentropy $\mathcal{I}$, and the total information I, which is constant under the unitary evolution. Moreover, since measurements tend to always increase $\mathcal{I}$, by bringing the system closer to the product state $|s_p\rangle$, and hence destroying the correlations, the evolution fol-

**An example of such hamiltonians can be found in the lectures of Lugiato[42]—it is used there to justify the master equation. See also discussion of measurement by Stenholm[43] and his distinction between T_1 and T_2 processes.

lowing the measurement is likely to decrease $\mathcal{I}$. An important problem which has not been treated here in sufficient detail is the problem of the division of the original system into the subsystems. I have assumed that such a division can be inferred for the system not interacting with the environment from the total hamiltonian that is from the H^S plus the hamiltonian coupling the system with the measuring device.

When the system is not perfectly isolated, its entropy increases due to the "leakage" of the information into the environment. This second process leaves the system in a true mixture, which can be calculated by tracing out states of the environment. It might appear that this second process could be made arbitrarily slow. After all, there is nothing in principle to prevent one from limiting, say, the rate of exchange of energy between the system and its environment. The quantity that is of concern to us here is, nevertheless, the information. To conserve it one must keep the state of the system, $|\Phi(t)\rangle$, uncorrelated with the state of the environment. That is, the total system-environment wave function $|\Theta\rangle$ it must be true at all times that:

$$|\Theta(t)\rangle = |\Phi(t)\rangle \otimes |\epsilon(t)\rangle \tag{5.7}$$

This may seem easy to achieve for the time intervals of the order of the relaxation time of the system. It is, however, doubtful whether such an idealization can be valid for periods as long as the Poincaré recurrence time. This in turn implies that the difficulty with the quantum version of Zermelo's paradox[41] disappears: The state of the system decays into a density matrix before it gets a chance to make a Zermelo "comeback".

To conclude this section let me restate its main points. According to the view proposed here entropy can be regarded as a negative of the information residing in the natural subsystems of the combined system. This structure is not arbitrary: It is presumably rigidly defined by the self-hamiltonian of the system with the measuring hamiltonian playing on additional role. The increase of entropy is caused by two distinct information transfers. First, there is a decrease of the negentropy $\mathcal{I}$ due to the buildup of the correlations. It is responsible for the entropy increase on the relaxation time scale. Second, there is a much slower buildup of the correlation between the system and its environment. It is responsible for the ultimate decay of the pure state into the density matrix. I believe that, combined, these two processes can explain both the relaxation and the persistence of the thermodynamic equilibria.

Amplification and Redundancy

The aim of the following discussion is to argue that in the absence of an exact pointer basis it is advantageous to make not one, but many copies of the same information. This can be achieved by using a collection of microscopic systems, which together constitute a "macroscopic" apparatus. The process of amplification facilitates the choice of the reliable basis of the pointer, that is, that basis which keeps the correlation with the measured system for a long time. (The exact pointer basis retains the record for an arbitrarily long time). This strategy of making many copies of the same message is known in the communication science as the strategy of using redundancy in order to increase reliability of stored messages.

Consider a simple model for a macroscopic apparatus: N two-state atoms. Let the first stage of the measurement of a spin be represented by a transition:

$$|\Phi_i\rangle = (a|\uparrow\rangle + b|\downarrow\rangle)\prod_{k=1}^{N}\otimes|+\rangle \longrightarrow$$

$$\longrightarrow a|\uparrow\rangle\prod_{k=1}^{N}\otimes|\pm\rangle_k + b|\downarrow\rangle\prod_{k=1}^{N}\otimes|\mp\rangle_k \tag{5.8}$$

In the absence of any particular symmetry in the apparatus-environment interaction the two states; $\prod\otimes|\pm\rangle_k$, $\prod\otimes|\mp\rangle_k$ are a natural choice for the reliable pointer basis of the apparatus. Of course, these two states do not span a complete basis in the 2^N dimensional Hilbert space of the apparatus. This is precisely the meaning of redundancy: the space of possible massages is much larger than the subspace actually employed to encode the result of the measurement. Now even in the presence of external perturbations there is a clear-cut strategy which yields a reliable distinction between spin $|\uparrow\rangle$ and $|\downarrow\rangle$. Moreover, it is also possible to distinguish between these two states of the spin by consulting only a subset of all N atoms. To see why, consider a small apparatus, consisting of just three recording atoms, and assume that the spin is initially in a state $(|\uparrow\rangle + |\downarrow\rangle)//2$. Then the first stage of the measurement:

$$|\Phi_i\rangle = |\odot\rangle\otimes|+\rangle\otimes|+\rangle\otimes|+\rangle \longrightarrow$$

$$\longrightarrow (|\uparrow\rangle\otimes|\pm\rangle\otimes|\pm\rangle\otimes|\pm\rangle + |\downarrow\rangle\otimes|\mp\rangle\otimes|\mp\rangle\otimes|\mp\rangle)/(\sqrt{2})^3 = |\Phi_f\rangle \tag{5.9}$$

Even if due to fluctuations or some inadequacies of the apparatus one of the atoms gets "flipped", the rule of a thumb "the spin was $|\uparrow\rangle$ if the majority of atoms was in the state $|\pm\rangle$" suffices to be correct. Moreover, if there are no disturbances and $|\Phi_f\rangle$ of Eq. (5.8), (5.9) remains unperturbed, it is sufficient to consult only a subset of N atoms to learn about the state of the spin. Of course, one can still rewrite $|\Phi_f\rangle$ by using any alternative set of the relative states. For example:

$$|\Phi_f\rangle = \{|\uparrow\rangle \otimes [|\pm\rangle \otimes |\pm\rangle \otimes |\pm\rangle] + |\downarrow\rangle \otimes [|\mp\rangle \otimes |\mp\rangle \otimes |\mp\rangle]\}/(\sqrt{2})^3$$

$$\{|\odot\rangle\otimes[(|+\rangle\otimes|+\rangle\otimes|+\rangle)+(|+\rangle\otimes|-\rangle\otimes|-\rangle)+(|-\rangle\otimes|+\rangle\otimes|-\rangle)+(|-\rangle\otimes|-\rangle\otimes|+\rangle)]+$$

$$|\otimes\rangle\otimes[(|-\rangle\otimes|-\rangle\otimes|-\rangle)+(|-\rangle\otimes|+\rangle\otimes|+\rangle)+(|+\rangle\otimes|-\rangle\otimes|+\rangle)+(|+\rangle\otimes|+\rangle\otimes|-\rangle)]\}/(\sqrt{2})^3$$

Now an even number of atoms in the state $|-\rangle$ implies relative state $|\odot\rangle$ while an odd number of $|-\rangle$ atoms imples the alternative $|\otimes\rangle$. This rule generalizes for the $\{|\odot\rangle, |\otimes\rangle\}$ spin basis to an arbitrary N. Thus, it is not difficult to see why, while there is nothing in principle which could keep one from distinguishing between any pair of the relative spin states, it is still easiest to tell $|\uparrow\rangle$ from $|\downarrow\rangle$. In particular, it will become successively more difficult to distinguish between the states $|\odot\rangle$ and $|\otimes\rangle$ and N becomes truly macroscopic, $N \sim 10^{23}$ atoms or so.

Redundancy is the reason why amplification is a good strategy for making reliable records. The reason why certain states seem a natural choice goes beyond redundancy: It contains also a basic assumption that the readout of the information contained in the apparatus will recognize atoms as distinct subsystems. Consequently, if the information contained in some part of the apparatus were to become obliterated, the remaining record would still suffice to distinguish between $|\uparrow\rangle$ and $|\downarrow\rangle$ states of the spin. This point of view in which apparatus is represented by a collection of distinct Hilbert spaces has been discussed, for instance, by Hepp[30], and more recently by Machida and Namiki[44], and Araki[45].

Pointer Basis and the Operational Interpretation of Quantum Theory

The aim of this last subsection is to consider the relevance of the environment-induced superselection rules in the context of the interpretation of quantum mechanics. The emergence of the pointer basis indicates, I shall

argue, that from the standpoint of the observer external to the system-apparatus object, the measurement yields a definite result for a definite observable only when, apart from the system and the apparatus, there is a third object, called here the environment. Presence of the environment is essential in making the choice between many to-be-measured observables.

In discussions of quantum measurements in which the apparatus is explicitly quantum (e.g., von Neumann's treatment) it was always tacitly taken for granted that the quantum measuring apparatus somehow contains the information determining which observable is going to be recorded. This distincion between various measuring procedures, so essential for Bohr's concept of complementarity and in the Copenhagen interpretation of quantum mechanics is not present for a truly quantum apparatus. To see why we return to the bit-by-bit "measurement" in the case of the perfect nonseparable correlation, Eq. (3.8a). There the apparatus-atom can always supply a definite and exact value of the angular momentum of the spin in any direction. Thus, one is forced to admit that the atom "knows" the component of the spin in an arbitrary direction with perfect accuracy. When the atom is considered to be a representative model of an apparatus (in disagreement with the Copenhagen interpretation of quantum theory) this last conclusion appears to be in a flagrant violation of the principle of complementarity: The apparatus-atom "knows" all about complementary components of the spin. Short of Copenhagen or Many World interpretations (to which we shall come back) there is only one way of avoiding a conflict between complementarity and nonseparability: One must recognize that while the two nonseparably correlated quantum systems have a complete information about each other, they contain no information about the outside Universe.

In particular, atom doesn't "know" what is the spin observable it has recorded. This information shall be provided by the correlation it will establish with some other system—with the environment. That is, the atom does know all that is to be known about the state of the spin, but this information cannot be expressed in terms of some absolute coordinate system. To see how the state of the measured system can be definite with respect to the apparatus, despite the fact that the system-apparatus combination remains in the state indefinite from the point of view of the external observer, let us investigate a system composed of two spin $\frac{1}{2}$ objects, described by the state

vector:

$$|\sigma\rangle = (|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2)/\sqrt{2} \tag{5.10}$$

Nothing can keep one from thinking of the first spin as of the measured system and of the second spin as of a quantum apparatus. After the first stage of the measurement, which has resulted in the wave function $|\sigma\rangle$, the state of the spin-system with respect to the spin- apparatus is quite definite: The (no. 1) spin-system always points in the direction which is opposite to the orientation of the (no. 2) spin-apparatus. This is a definite, "coordinate-independent" statement. Thus, from the point of view of the spin-apparatus, the measurement has already yielded a definite result. However, from the point of view of an external observer the measurement has not been completed: The state of the spin remains indefinite with respect to the state of the coordinate system in which this observer describes physics. The result of the spin-spin measurement, already definite from the viewpoint of the spin-apparatus shall acquire definiteness with respect to the external observer only when the apparatus and the observer will become correlated. Two distinct possibilities arise here. First, the observer may consult the apparatus. Then the system-apparatus combination will enter into a state that is definite with respect to this observer, and, moreover, the observer will become aware of the outcome of the measurement. The second possibility may arise when the apparatus enters into a correlation with the environment. Then the state of the apparatus with respect to the environment becomes definite, and if the environment remains in a definite state with respect to the observer, the state of the apparatus may as well become definite with respect to the observer. This does not imply that the observer knows the outcome (see discussion in the first subsection of this section).

The key idea of the proposed above reinterpretation of quantum mechanics is the operational definition of the outcome of measurement. The states of any physical system are defined only with respect to the other systems with which the system in question is actually correlated. In this sense operational interpretation of quantum mechanics is very close to the Relative State interpretation of Everett. The state of the system is defined with respect to the state of the observer. The new element, present here but not in the original Everett's proposal is that the distinct states of the observer should not be under certain circumstances thought of as distinguishable by this observer. The motivation for such a reinterpretation of quantum theory comes not from the dissatisfaction with the stochastic nature of quantum predictions. Rather, it arises from the following dilemma: The observer (apparatus) presumably "knows" the outcome of the measurement as soon as the measurement is performed. Yet, this definiteness of the outcome is in conflict with the principle of superposition, as it can be manifested by the reversibility of the hamiltonian evolution: Some "higher order" observer, for which the first observer (an apparatus, Schrödinger's cat, Wigner's friend, $\cdots$) is just a term in the wave function can presumably consult the first observer and learn this way about some definite outcome of the measurement, or can reverse the evolution of the combined apparatus-system object. Such time reversal can bring back the initial state only if the complete superposition, containing all the potential outcomes of the experiment that could have been registered by the apparatus is still present when the reversal procedure is initiated. Thus, on one hand, the apparatus is supposed to learn about the definite outcome, while on the other hand it is supposed to be forced into a superposition corresponding to all the conceivable outcomes. Can one resolve this contradiction? The Copenhagen interpretation avoids it by dividing systems present in the Universe into two distinct classes; microscopic, and therefore quantum, and macroscopic, and therefore classical. The Relative State (also known as the "Many World") interpretation bypasses the problem by saying that the second observer "splits" already when the first one splits. Is there any other, less drastic solution of this dilemma? The only other proposal not yet explored is the extension of the approach discussed above on the example of two correlated spins. There the outcomes that were distinct from the standpoint of the external observer were indistinguishable to the first "observer"; spin-apparatus. Can this explanation be employed to resolve all the apparent paradoxes arising in quantum theory of measurements?

Our discussion certainly does not go far enough to yield a definite answer to this question. It may, in particular, be possible that the operational point of view, in which all the observables are defined "with respect to" rather than in absolute terms will lead to predictions testably different from those of Copenhagen interpretation. Let me, moreover, stress that the operational interpretation of quantum measurements is close to the position adopted by Bohr, who insists that quantum theory can be applied only by macroscopic observers using macroscopic apparata. Macrophysics is regarded by Bohr as more fundamental than microphysics, for it is within the realm of classical physics where "a discrimination between different experimental procedures which allows for the unambiguous use of the complementary classical concepts"†† occurs. In the operational approach to the process of measurement similarly as in the Copenhagen interpretation it is the task of the experimental arrangement to define what are the distinct recorded properties. Only after the to-be-measured property has been operationally defined (e.g. by the direction of the field gradient in the Stern-Gerlach apparatus) can a quantum phenomenon be brought to closure: Via collapse of the wavepacket the quantum system shall assume a definite eigenstate of the measured observable. It is, however, absolutely crucial that this observable, the "property" must be first, prior to the collapse, defined by the experimental arrangement. In this sense the property does not belong entirely to the measured quantum system; it is rather forced upon it by the act of measurement. The key objection against the Copenhagen interpretation comes from the fact that it demands a parallel existence of two distinct domains: macro and microscopic. This objection would not apply to the operational interpretation. (Of course there may be many, much more serious objections which I have not yet discovered.) Operational interpretation attempts to give a unified point of view, where the whole Universe is described by the same theory. It is worth noting that the same goal of a unified description of physics has been adopted by the Many World (alias Many Universes, alias Relative State) interpretation.[16,36,40] There, each time an interaction occurs between two subsystems of the Universe, the whole Universe branches out into as many new Universes as there are outcomes of the measurement. The key reason for this splitting comes from the assumption of existence of abstract observables, which do not have to be realized operationally by means of

††This quotation is taken from Bohr's reply[46] to the Einstein, Podolsky and Rosen argument.[4]

an experimental arrangement. This point of view encounters several difficulties. First of all, this splitting is unobservable, and hence unphysical (but see reference[40]). Moreover, it is difficult to accept the idea that our consciousness splits due to a collision of two electrons in some yet-to-be discovered quasar on the outskirts of the Universe. Furthermore, there are often difficulties in determining branches into which the Universe should split. This ambiguity is due to the freedom of choice of the two relative bases.[40] Also, quite often the Universes in which the splitting occurs must recombine to accommodate interference experiments. And last, but not least, adopting Many World interpretation has not led to any new results in, for example, quantization of General Relativity, as it was originally anticipated.[16,36] Above brief discussion can do justice neither to the Many World interpretation nor to the difficulties it encounters. Nevertheless one is led to believe that, like the Copenhagen interpretation the Many World interpretation conceals rather than explains problems inherent in quantum formalism.

The operational interpretation explored preliminarily above asserts that there is no wave function for the whole Universe. Every observer has his own description (wave-function, or, more generally, density matrix) of the physical systems around him. This description is always expressed in a language appropriate for that observer, e.g. the measuring spin $\frac{1}{2}$ particle describes the rest of the world (all he can see) like this: $|$anti-aligned with me$\rangle$. It is quite conceivable that one could develop this very preliminary proposal into a consistent interpretation of quantum theory. It is also possible, that by insisting on the observables that are defined only "with respect to" rather than in some absolute sense one may arrive at a theory that is significantly (testably) different from quantum mechanics. Preliminary investigation of the basic ideas of the operational interpretation does not yet allow to distinguish between these two alternatives.

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